LIBERTY PAPER SET

STD. 10 : Mathematics (Basic) [N-018(E)]

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 7**

Section-A

1. (B) 2 **2.** (A) $c = a$ **3.** (C) $\frac{-14}{3}$ **4.** (B) –12 **5.** (B) 0° **6.** (D) 2 **7.** 360 **8.** 2 **9.** $\frac{4}{11}$ **10.** Increases **11.** the tangent of the circle **12.** 3 **13.** True **14.** False **15.** True **16.** False **17.** $k = 10$ **18.** 16 cm **19.** 480 tickets **20.** 6.25 **21.** $\frac{\pi R^2 P}{360^\circ}$ **22.** 60 cm **23.** (b) $4\pi r^2$ **24.** (c) $2\pi rh$

Section-B

25. $x^2 + 5x - 24 = 0$

$\therefore x^2 + 8x - 3x - 24 = 0$

$\therefore x(x + 8) - 3(x + 8) = 0$

$\therefore (x + 8)(x - 3) = 0$

$\therefore x + 8 = 0$ and $x - 3 = 0$

$\therefore x = -8$ and $x = 3$

Sum of the zeros $= -8 + 3 = -5 = -\frac{5}{1} = -\frac{b}{a} = -\frac{\text{co efficient of } x}{\text{co efficient of } x^2}$

Product of the zeros $= (-8)(3) = -24 = \frac{-24}{1} = \frac{c}{a} = \frac{\text{costant term}}{\text{co efficient of } x^2}$

26. Let the quadratic polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$\therefore \alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$ and $\alpha\beta = -6 = \frac{-6}{1} = \frac{c}{a}$

$\therefore$ a = 1, b = – 1, c = – 6

So, one quadratic polynomial which fits the given condition is $x^2 - x - 6$. You can check that any other quadratic polynomial that fits these conditions will be of the form $k(x^2 - x - 6)$, where k is real.

27. Here $x = -3$,

Now, $x^2 + 3(k + 2)x - 9 = 0$

$\therefore (-3)^2 + 3(k + 2)(-3) - 9 = 0$

$\therefore 9 - 9k - 18 - 9 = 0$

$\therefore 9k = -18$

$\therefore k = -2$

28. Here, $a = 5$, $d = 11 - 5 = 6$

$\therefore a_{101} = a + 100d$

$\therefore a_{101} = 5 + 100(6)$

$\therefore a_{101} = 5 + 600$

$\therefore a_{101} = 605$

29. Here, $a = -5$, $d = 2$, $n = 6$

$S_n = \frac{n}{2}\ [2a + (n-1)\ d]$

$\therefore\ S_6 = \frac{6}{2}\ [2\ (-5) + (6-1)]^2$

$= 3\ (-10 + 10)$

$= 3\ (0) = 0$

30. Suppose, the point P (x, y) is equidistant from A (3, 6) and B (–3, 4).

$\therefore$ PA = PB

$\therefore$ $PA^2 = PB^2$

$\therefore$ $(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$

$\therefore$ $x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$

$\therefore$ $-6x - 12y + 36 = 6x - 8y + 16$

$\therefore$ $-6x - 12y + 36 - 6x + 8y - 16 = 0$

$\therefore$ $-12x - 4y + 20 = 0$

$\therefore$ $3x + y - 5 = 0$

Hence, the relation between x & y is $3x + y - 5 = 0$.

31. Co-ordinates from the midpoint of the diagonal AC = Co-ordinates from the midpoint of the diagonal BD

$\therefore\ \left(\frac{1+3}{2}, \frac{2-4}{2}\right) = \left(\frac{2+x}{2}, \frac{1+y}{2}\right)$

$\therefore\ (2, -1) = \left(\frac{2+x}{2}, \frac{1+y}{2}\right)$

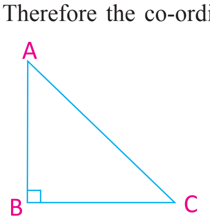
$\therefore\ 2 = \frac{2+x}{2}\quad , \quad -1 = \frac{1+y}{2}$

$\therefore\ 2 + x = 4\quad , \quad 1 + y = -2$

$\therefore\ x = 2\quad , \quad y = -3$

Therefore the co-ordinates of D (x, y) = D (2, –3)

32.



In ΔABC, = ∠B 90° and = ∠C = θ

$cosec\ \theta = \frac{13}{5}$

$\therefore\ \frac{AC}{AB} = \frac{13}{5}$

$\therefore\ \frac{AC}{13} = \frac{AB}{5} = k$, k is positive

$\therefore$ AC = 13 k, AB = 5k

According to Pythagoras theorem :

$BC^2 = AC^2 - AB^2$

$\therefore\ BC^2 = (13\ k)^2 - (5\ k)^2$

$\therefore\ BC^2 = 169\ k^2 - 25\ k^2$

$\therefore\ BC^2 = 144\ k^2$

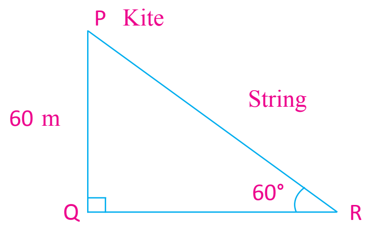
$\therefore\ BC = 12\ k$

$\therefore\ \tan\theta = \frac{AB}{BC} = \frac{5\,k}{12\,k} = \frac{5}{12}$ and

$\cos\theta = \frac{BC}{AC} = \frac{12\,k}{13\,k} = \frac{12}{13}$

33. $\frac{2\tan 60^\circ}{1+\tan^2 60^\circ} = \frac{2\sqrt{3}}{1+(\sqrt{3})^2} = \frac{2\sqrt{3}}{1+3} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

34.



Here, P represents the position of the kite and the string is tied to point R on the ground.

As well as PQ is the height of the kite from the ground.

In Δ PQR, $\angle Q = 90°$, $\angle R = 60°$ and PQ = 60 m

$\therefore\ \sin R = \frac{PQ}{PR}$

$\therefore\ \sin 60° = \frac{60}{PR}$

$\therefore\ \frac{\sqrt{3}}{2} = \frac{60}{PR}$

$\therefore\ PR = \frac{120}{\sqrt{3}} = 40\sqrt{3}$ m

Hence, the length of the string $40\sqrt{3}$ m

35. $\frac{v_1}{v_2} = \frac{\frac{4}{3}\pi r_1^{\,3}}{\frac{4}{3}\pi r_2^{\,3}} = \frac{64}{27}$

$\therefore\ \frac{r_1^{\,3}}{r_2^{\,3}} = \frac{64}{27}$

$\therefore\ \frac{r_1}{r_2} = \frac{4}{3}$

$\therefore\ \frac{r_1^{\,2}}{r_2^{\,2}} = \frac{16}{9}$

$\therefore\ \frac{A_1}{A_2} = \frac{4\,\pi r_1^{\,2}}{4\,\pi r_2^{\,2}} = \frac{r_1^{\,2}}{r_2^{\,2}} = \frac{16}{9} = 16 : 9$

36. $r = 2.1$ cm, $h = 5$ m

Volume of tank = $\pi r^2 h$

$$= \frac{22}{7} \times 2.1^2 \times 5$$

$$= \frac{22 \times 2.1 \times 2.1 \times 5}{7}$$

$$= \frac{22 \times 21 \times 21 \times 5}{7 \times 10 \times 10}$$

$$= \frac{11 \times 2 \times 21 \times 7 \times 3 \times 5}{7 \times 5 \times 2 \times 10}$$

$$= \frac{11 \times 21 \times 3}{10}$$

$= 69.3 \text{ m}^3$

$\therefore$ Capacity = 69.3 × 1000 lit. = 69300 lit.

37.

Marks obtained (x_i)	Number of students (f_i)	$f_i x_i$
10	1	10
20	1	20
36	3	108
40	4	160
50	3	150
56	2	112
60	4	240
70	4	280
72	1	72
80	1	80
88	2	176
92	3	276
95	1	95
total	$\Sigma f_i = 30$	$\Sigma f_i x_i = 1779$

$$\therefore \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1779}{30} = \frac{593}{10} = 59.3$$

$\therefore \bar{x} = 59.3$

38. By the method of elimination :

$x + y = 5$ ...(1)

$2x - 3y = 4$ ...(2)

Multiply equation (1) by 3 and equation (2) by 1 and add them

$3x + 3y = 15$

$2x - 3y = 4$

$\therefore 5x = 19$

$\therefore x = \frac{19}{5}$

Put $x = \frac{19}{5}$ in equation (1)

$x + y = 5$

$\therefore \frac{19}{5} + y = 5$

$\therefore y = 5 - \frac{19}{5}$

$\therefore y = \frac{25-19}{5}$

$\therefore y = \frac{6}{5}$

The solution of the equation : $x = \frac{19}{5}, y = \frac{6}{5}$

39. Suppose, Meena received the number ₹ 50 notes be x and ₹ 100 notes be y.

According to the first condition,

$50x + 100y = 2000$

$\therefore x + 2y = 40$ ...(1)

According to the second condition,

$x + y = 25$ ...(2)

Subtracting equation (1) & (2)

$$\begin{array}{r} x + 2y = 40 \\ x + y = 25 \\ - \quad - \quad\quad - \\ \hline \end{array}$$

$\therefore y = 15$

Put $y = 15$ in equation (2)

$x + y = 25$

$\therefore x + 15 = 25$

$\therefore x = 10$

Hence, Meena has 10 notes ₹ 50 and 15 notes of ₹ 100

40. $a = 5$, $an = 95$, $d = 5$

$an = a + (n - 1)\ d$

$\therefore 95 = 5 + (n - 1)\ 5$

$\therefore 95 - 5 = (n - 1)\ 5$

$\therefore \frac{90}{5} = n - 1$

$\therefore n - 1 = 18$

$\therefore n = 19$

$S_n = \frac{n}{2}$ (a + an)

$\therefore S_{19} = \frac{19}{2}(5 + 95)$

$\therefore S_{19} = \frac{19}{2} \times 100$

$\therefore S_{19} = 950$

41. Let the ratio on which the y-axis point P $(0, y)$ divides the line segment joining the point A $(5, -6)$ and B $(-1, -4)$ be $m_1 : m_2$.

The coordinates of

$$P = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$\therefore (0, y) = \left(\frac{m_1(-1) + m_2(5)}{m_1 + m_2}, \frac{m_1(-4) + m_2(-6)}{m_1 + m_2}\right)$$

$$\therefore (0, y) = \left(\frac{-m_1 + 5m_2}{m_1 + m_2}, \frac{-4m_1 - 6m_2}{m_1 + m_2}\right)$$

$$\therefore 0 = \frac{-m_1 + 5m_2}{m_1 + m_2}, \quad y = \frac{-4m_1 - 6m_2}{m_1 + m_2}$$

$\therefore -m_1 + 5m_2 = 0$ $\qquad$ $\therefore y = \frac{-4(5) - 6(1)}{5 + 1}$

$\therefore -m_1 = -5m_2$ $\qquad$ $\therefore y = \frac{-20 - 6}{6}$

$\therefore m_1 = 5m_2$ $\qquad$ $\therefore y = \frac{-26}{6}$

$\therefore \frac{m_1}{m_2} = \frac{5}{1}$ $\qquad$ $\therefore y = -\frac{13}{3}$

$\therefore m_1 : m_2 = 5 : 1$

Therefore, the required ratio is 5 : 1. The point of intersection is $\left(0, -\frac{13}{3}\right)$.

42. Let, P (x, y) be the point which divides the line segment joining the points A $(4, -3)$ and B $(8, 5)$ in the ratio 3:1 internally.

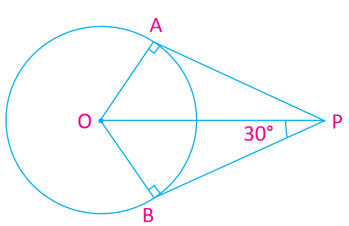
$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \qquad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\therefore x = \frac{3(8) + 1(4)}{3 + 1}, \qquad y = \frac{3(5) + 1(-3)}{3 + 1}$$

$\therefore x = 7,$ $\qquad$ $y = 3$

Therefore, $(7, 3)$ is the required point.

43.



OB ⊥ PB So, ∠OBP = 90°

In Δ OBP; ∠BOP + ∠OBP + ∠OPB = 180°

∴ ∠BOP + 90° + 30° = 180°

∴ ∠BOP + 120° = 180°

∴ ∠BOP = 180° – 120°

∴ ∠BOP = 60°

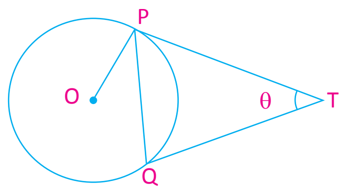
OP is bisector of ∠AOB.

∴ ∠AOB = 2∠BOP

∴ ∠AOB = 2 × 60°

∴ ∠AOB = 120°

44.



We are given a circle with centre O, an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact.

Suppose, $\angle PTQ = \theta$

Now, TP = TQ (theorem 10.2)

So, Δ TPQ is an isosceles triangle.

$$\therefore \angle TPQ = \angle TQP = \frac{1}{2}(180° - \angle PTQ)$$

$$= \frac{1}{2}(180° - \theta)$$

$$= 90° - \frac{1}{2}\theta$$

Now, $\angle OPT = 90°$ (theorem 10.1)

$$\therefore \angle OPQ = \angle OPT - \angle TPQ$$

$$= 90° - \left(90° - \frac{1}{2}\theta\right)$$

$$= 90° - 90° + \frac{1}{2}\theta$$

$$= \frac{1}{2}\theta$$

$$\therefore \angle OPQ = \frac{1}{2}\angle PTQ$$

$$\therefore \angle PTQ = 2\angle OPQ$$

45. Here, the class length is not the same so we will use the method of deviation on from $a = 17$ and $h = 10$.

Number of days (class)	Number of students (f_i)	x_i	u_i	$f_i u_i$
0 – 6	11	3	– 1.4	– 15.4
6 – 10	10	8	– 0.9	– 9.0
10 – 14	7	12	– 0.5	– 3.5
14 – 20	4	17 = a	0	0
20 – 28	4	24	0.7	2.8
28 – 38	3	33	1.6	4.8
38 – 40	1	39	2.2	2.2
Total	40	–	–	– 18.1

$$\text{Mean } \bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$\therefore \bar{x} = 17 + \frac{-18.1 \times 10}{40}$$

$$\therefore \bar{x} = 17 - 4.525$$

$$\therefore \bar{x} = 12.475$$

$$\bar{x} = 12.48 \text{ (Approx)}$$

Hence, mean No. of days for which a student was absent is 12.48 days.

46. Total number of possible outcomes on dice = 6

(i) Suppose event X faces with letter A on it.

$$\therefore P(X) = \frac{\text{Total number of faces with letter A}}{\text{Total number of outcomes}}$$

$$\therefore P(X) = \frac{2}{6} = \frac{1}{3}$$

(ii) Suppose event Y faces with letter D.

$$\therefore P(Y) = \frac{\text{Total number of faces with letter D}}{\text{Total number of outcomes}}$$

$$\therefore P(Y) = \frac{1}{6}$$

(iii) Suppose event Z faces with letter is vowel.

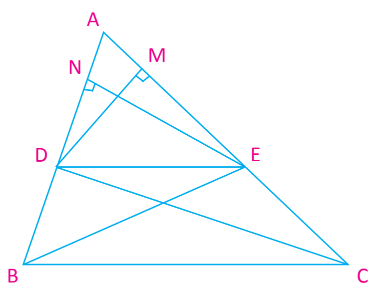
$$\therefore P(Z) = \frac{\text{Total number of faces with letter vowel}}{\text{Total number of outcomes}}$$

$$\therefore P(Z) = \frac{3}{6} = \frac{1}{2}$$

47. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given: In ABC, a line parallel to side BC intersects AB and AC at D and E respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$



Proof: Join BE and CD and also draw DM $\perp$ AC and EN $\perp$ AB.

Then, ADE $= \frac{1}{2} \times AD \times EN$,

BDE $= \frac{1}{2} \times DB \times EN$,

ADE $= \frac{1}{2} \times AE \times DM$ and

DEC $= \frac{1}{2} \times EC \times DM$.

$$\therefore \frac{ADE}{BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \qquad ...(1)$$

$$\text{and } \frac{ADE}{DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \qquad ...(2)$$

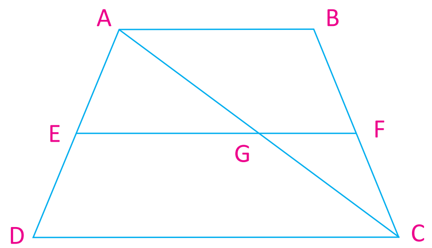
Now, Δ BDE and Δ DEC are triangles on the same base DE and between the parallel BC and DE.

then, BDE = DEC ...(3)

Hence from eq^n. (1), (2) and (3),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

48.



Let us join AC to intersect EF at G.

AB || DC and EF || AB (Given)

∴ EF || DC

∴ EG || DC and GF || AB (∵ AB || DC)

Now, in ΔADC, EG || DC

$$\therefore \frac{AE}{ED} = \frac{AG}{GC} \quad \text{(Theorem 6.1)} \quad ...(1)$$

Similarly from, ΔCAB, GF || AB

$$\therefore \frac{CG}{AG} = \frac{CF}{BF}$$

$$\therefore \frac{AG}{CG} = \frac{BF}{CF} \quad ...(2)$$

As per eqn. (1) and (2),

$$\frac{AE}{ED} = \frac{BF}{CF}$$

49. Suppose, the first positive integer = x,

The second consecutive positive integer = $x + 1$

According to condition,

$(x)^2 + (x + 1)^2 = 365$

$\therefore x^2 + x^2 + 2x + 1 = 365$

$\therefore 2x^2 + 2x + 1 - 365 = 0$

$\therefore 2x^2 + 2x - 364 = 0$

$\therefore x^2 + x - 182 = 0$

$\therefore x^2 + 14x - 13x - 182 = 0$

$\therefore x(x + 14) - 13(x + 14) = 0$

$\therefore (x - 13)(x + 14) = 0$

$\therefore x - 13 = 0$ OR $x + 14 = 0$

$\therefore x = 13$ OR $x = -14$

But $x = -14$ is not positive integer, therefore, required two consecutive positive integers will be 13 and 14.

50. (i) $a_2 = a + d = 13$ ...(1)

$a_4 = a + 3d = 3$ ...(2)

Subtracting equation (1) from (2), we get,

$(a + d) - (a + 3d) = 13 - 3$

$\therefore a + d - a - 3d = 10$

$\therefore -2d = 10$

$\therefore d = -5$

From (1) putting the value of $d = -5$,

$a + d = 13$

$\therefore a - 5 = 13$

$\therefore a = 18$

$\therefore a_3 = a + 2d = 18 + 2(-5) = 18 - 10 = 8$

Ans. : $\boxed{18}$, $\boxed{8}$

(ii) $a_2 = a + d = 38$ ...(1)

$a_6 = a + 5d = -22$ ...(2)

Subtracting equation (1) from (2), we get,

$(a + d) - (a + 5d) = 38 - (-22)$

$\therefore a + d - a - 5d = 38 + 22$

$\therefore -4d = 60$

$\therefore d = -15$

From equation (1) putting the value of $d = -15$

$a + d = 38$

$\therefore a - 15 = 38$

$\therefore a = 53$

$\therefore a_3 = a + 2d = 53 + 2(-15) = 53 - 30 = 23$

$\therefore a_4 = a + 3d = 53 + 3(-15) = 53 - 45 = 8$

$\therefore a_5 = a + 4d = 53 + 4(-15) = 53 - 60 = -7$

Ans. : $\boxed{53}$, $\boxed{23}$, $\boxed{8}$, $\boxed{-7}$

51. Here maximum class frequency is 18 which belongs to modal class 4000 – 5000.

$\therefore$ l = lower limit of modal class = 4000

h = class size = 1000

f_1 = frequency of modal class = 18

f_0 = frequency of class preceding the modal class = 4

f_2 = frequency of class succeeding modal class = 9

$$\text{Mode Z} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\therefore Z = 4000 + \left(\frac{18 - 4}{2(18) - 4 - 9}\right) \times 1000$$

$$\therefore Z = 4000 + \frac{14 \times 1000}{36 - 4 - 9}$$

$$\therefore Z = 4000 + \frac{14000}{23}$$

$$\therefore Z = 4000 + 608.7$$

$$\therefore Z = 4608.7$$

So, mode of given data is 4608.7.

52.

Monthly unit (usage)	65 – 85	85 – 105	105 – 125	125 – 145	145 – 165	165 – 185	185 – 205
No. of customer	04	05	–	20	–	08	04

Monthly unit (usage)	No. of customer (f)	(cf)
65 – 85	4	4
85 – 105	5	9
105 – 125	x	$9 + x$
125 – 145	20	$29 + x$
145 – 165	y	$29 + x + y$
165 – 185	8	$37 + x + y$
185 – 205	4	$41 + x + y$
	$n = 41 + x + y$	

Here, median = 137

$\Sigma f_i = n = 68 = 41 + x + y$

Median - class = 125 – 145

l = lower limit of median class = 125

cf = cumulative frequency of class preceding the median class = $9 + x$

f = frequency of median class = 20

$\frac{n}{2} = \frac{68}{2} = 34$

h = class size = 20

$$\text{Median M} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\therefore 137 = 125 + \left(\frac{34 - (9 + x)}{20}\right) \times 20$$

$\therefore 137 - 125 = 34 - 9 - x$

$\therefore 12 = 25 - x$

$\therefore x = 25 - 12$

$\therefore x = 13$

Now, $n = \Sigma x_i = 41 + x + y$

$\therefore 68 = 41 + 13 + y$

$\therefore 68 = 54 + y$

$\therefore y = 68 - 54$

$\therefore y = 14$

Thus, the number of customers with 105 to 125 and 145 to 165 unit usage is 13 and 14 respectively.

53. Here total number of students = 100

(i) Number of students getting more than 40 marks = 2 + 1 = 3

$$\text{Probability} = \frac{\text{Numbers of Students getting more than 40 marks}}{\text{Total number of students}}$$

$$= \frac{3}{100}$$

$$= 0.03$$

(ii) Number of students getting less than 30 marks = 6 + 20 + 24 + 28 = 78

$$\text{Probability} = \frac{\text{Numbers of Students getting less than 30 marks}}{\text{Total number of students}}$$

$$= \frac{78}{100}$$

$$= 0.78$$

(iii) $$\text{Probability} = \frac{\text{Numbers of Students getting 25 marks}}{\text{Total number of students}}$$

$$= \frac{20}{100}$$

$$= 0.2$$

(iv) $$\text{Probability} = \frac{\text{Numbers of Students getting 33 marks}}{\text{Total number of students}}$$

$$= \frac{15}{100}$$

$$= 0.15$$

54. Total result is 8. (HHH, HTH, HHT, HTT, THH, THT, TTH, TTT)

(i) Suppose event A is get at least two Heads.

(HHH, HHT, HTH, THH = 4)

$\therefore$ P (A) = $\frac{4}{8} = \frac{1}{2}$

(ii) Suppose event B is get exactly two Heads.

(HTH, HHT, THH = 3)

$\therefore$ P (B) = $\frac{3}{8}$

(iii) Suppose event C is number of Heads are more than tails.

(HTH, HHH, HHT, THH = 4)

$\therefore$ P (C) = $\frac{4}{8} = \frac{1}{2}$

(iv) Suppose event D is same result all times.

(three Heads of three tails, HHH, TTT = 2)

$\therefore$ P (D) = $\frac{2}{8} = \frac{1}{4}$